Appendix B: Reflection and Transmission of Light from Multilayer Films

Abstract: I review the electromagnetic theory of reflection and transmission of light from multilayered films of homogeneous nonmagnetic linear isotropic media. A derivation of a lesser-known transfer matrix method for calculating reflectivities and transmittivities is included; I have found this formulation to be particularly convenient for analytical calculations of reflectivity differences and computer numerical calculations.

B.1 Introduction

The main chapters of this dissertation are concerned with the use of OI-RD microscopes to detect chemical reactions in films of biomolecules that are immobilized on a solid substrate. Molecular events during a reaction, such as binding of reactants or conformational changes, result in a modification of the macroscopic linear optical properties of the film. In particular, the thickness and complex index of refraction of the film change. At oblique incidence, the reflectivities for s- and p-polarized light change disproportionately in response to the modification. OI-RD microscopes are designed to directly measure such disproportionate changes in the reflectivities. The relationships between the linear optical properties of a multilayer film system and the s- and p-polarized reflectivities are derived in this appendix. A Mathematica package implementation of these equations is listed in Appendix H. In Appendix C, these relations are used to compute reflectivity differences induced by changes in the optical properties of multilayer film systems. The means by which OI-RD microscopes measure reflectivity differences are discussed in Appendix D.

B.2 Plane Waves in Multilayer Films

The films and substrates of primary interest in this dissertation are linear isotropic media such as glass, randomly oriented organic macromolecules (e.g. proteins and DNA), and nonmagnetic metals (e.g. gold). In Appendix A, the theory of reflection and transmission of monochromatic plane waves from an interface between linear isotropic media was derived. Here, the concepts of reflectivity and transmittivity will be generalized to films composed of \( M \) parallel planar layers of linear isotropic media, as depicted in Figure B.1. It is assumed that the index of refraction changes abruptly (step-wise) at each interface and that the \( m \)-th layer has a thickness \( d_m \) and a homogeneous index of refraction \( \tilde{n}_m \). The multilayer film is bound on either side by semi-infinite media. The refractive indices of the film layers and transmission medium...
will be treated as complex numbers, allowing these media to be either transparent or absorbing. However, the incident medium is assumed to be transparent so that a uniform incident plane wave from a distant source can reach the film.

At each interface, Maxwell’s equations require continuity of the tangential component of the electric field. Similar to the case of a single interface, these boundary conditions require that a “forward” propagating and “back” propagating plane wave be present in each layer. Here “forward” means the sense of propagation is from the incident medium side to the transmission medium side of the layer and “back” means the opposite. In the incident medium, the forward propagating wave is the incident plane wave and the back propagating wave is the reflected plane wave; in the transmission medium, only the forward propagating transmitted wave is present. The boundary conditions also require that the angular frequencies of all the waves be the same, the complex wave vectors of each plane wave lie in a common plane (the plane of incidence) and that

\[ n_i \sin \theta_i = \bar{n}_i \sin \bar{\theta}_i = \ldots \bar{n}_M \sin \bar{\theta}_M = \bar{n}_{M+1} \sin \bar{\theta}_{M+1} , \]

(B.1)

where the complex “angles” \( \bar{\theta}_i \) and complex indices of refraction determine the complex wave vectors in the manner discussed in Appendix A. Furthermore, the complex angles for the forward and back propagating waves have the same value but different directions, as illustrated in Figure B.1. The complex angles can be used to decompose the fields into s-polarized (perpendicular to the plane of incidence) and p-polarized (parallel to the plane of incidence) components. These components are linearly independent of each other for reflection and transmission from isotropic media. That is, if the incident wave is s-polarized, then all of the forward and back propagating fields in the film layers will be s-polarized; likewise for p-polarization. The s-polarization reflectivity of the film is defined as

\[ R_s \equiv \frac{\bar{E}_s^r}{\bar{E}_s^i} , \]

and the s-polarization transmittivity is defined as

\[ T_s \equiv \frac{\bar{E}_s^t}{\bar{E}_s^i} , \]

(B.3)

where \( \bar{E}_s^i \) and \( \bar{E}_s^r \) are the s-polarization electric field phasors for the incident and reflected plane waves at the 1st interface and \( \bar{E}_s^t \) is the s-polarization electric field phasor of the transmitted plane wave at the \((M+1)\)-th interface. In addition, it is implicitly assumed that the electric field phasors are evaluated at identical x-y coordinates in the planes of the interfaces. Likewise, for p-polarization the reflectivity and transmittivity are defined as
The aim of the remaining sections in this appendix is to develop methods for calculating these transmittivities and reflectivities given the angular frequency (or equivalently, the vacuum wavelength) and incidence angle of the incident plane wave and the thicknesses and refractive indices of the film layers.

**B.3 Single Layer Films**

Single layer films are the simplest to calculate. Most optics textbooks provide a derivation for this situation using the method of summing partial waves [1-5]. This derivation is repeated in this section. When an incident plane wave strikes the first interface a partial wave is reflected and transmitted. The first transmitted wave proceeds to the second interface where another partial wave is reflected and transmitted and so forth *ad infinitum*. The partial waves and their corresponding electric field phasors are illustrated in Figure B.2. Assuming that the film thickness is much less than the lateral extent of the film (or for real-world finite beams, that the film thickness is much less than the diameter of the beam), no single partial wave can be isolated from the others. Therefore, it is the total reflected wave that is observed, which is the sum of all back propagating partial waves in the incident medium. The same reasoning applies to the total transmitted wave composed of all forward propagating partial waves in the transmission medium.

The phase and amplitude of the electric field change upon transmission or reflection from an interface. The ratio of the electric field phasors of the incident, reflected, and transmitted partial waves at each interface are given by the Fresnel equations, which were derived in Appendix A. For a wave incident from medium $a$ onto medium $b$ (denoted $ab$), the Fresnel equations for the reflectivities and transmittivities are:

\[
\tilde{r}_p^{(ab)} = \frac{\tilde{n}_p \cos \tilde{\theta}_s - \tilde{n}_i \cos \tilde{\theta}_a}{\tilde{n}_i \cos \tilde{\theta}_s + \tilde{n}_p \cos \tilde{\theta}_a}, \tag{B.6}
\]

\[
\tilde{r}_s^{(ab)} = \frac{\tilde{n}_s \cos \tilde{\theta}_s - \tilde{n}_i \cos \tilde{\theta}_a}{\tilde{n}_i \cos \tilde{\theta}_s + \tilde{n}_s \cos \tilde{\theta}_a}, \tag{B.7}
\]

\[
\tilde{t}_s^{(ab)} = \frac{2\tilde{n}_s \cos \tilde{\theta}_s}{\tilde{n}_i \cos \tilde{\theta}_s + \tilde{n}_s \cos \tilde{\theta}_a}, \tag{B.8}
\]

\[
\tilde{t}_p^{(ab)} = \frac{2\tilde{n}_p \cos \tilde{\theta}_s}{\tilde{n}_i \cos \tilde{\theta}_s + \tilde{n}_p \cos \tilde{\theta}_a}. \tag{B.9}
\]
Furthermore, $ab$ and $ba$ reflectivities and transmittivities are connected by the Stokes relations (also derived in Appendix A)

$$\tilde{r}_{sp}^{(ab)} = -\tilde{r}_{sp}^{(ba)}$$  \hspace{1cm} (B.10)

and

$$\left(\tilde{r}_{sp}^{(ab)}\right)^2 + \tilde{t}_{sp}^{(ab)}\tilde{r}_{sp}^{(ba)} = 1,$$  \hspace{1cm} (B.11)

where $s \parallel p$ indicates the equations hold for either $s$- or $p$-polarized light. In addition to the effects of the interface, the wave amplitudes and phases also change as the waves propagate through each layer. Consider the constant phase wave fronts labeled A and B in Figure B.2. Assuming the partial wave propagating from A to B is a harmonic plane wave of the form $\tilde{E}(r,t) = \tilde{E}_0 \exp\left(i\left(\tilde{k} \cdot r - \omega t\right)\right)$, the electric fields at the top and bottom of the layer are proportional

$$\exp(i\tilde{\gamma})\tilde{E}(x\hat{x} + y\hat{y},t) = \tilde{E}(x\hat{x} + y\hat{y} + d\hat{z},t)$$  \hspace{1cm} (B.12)

where

$$\tilde{\gamma} = \tilde{k} \cdot d, \hat{z} = 2\pi \hat{n}, \cos \tilde{\theta} \frac{d}{\lambda}$$  \hspace{1cm} (B.13)

and $\lambda = 2\pi c/\omega$ is the vacuum wavelength of the plane wave. Using the interface reflectivities (B.6)-(B.9) and the propagation factor (B.12), the electric field phasors of each of the reflected and transmitted partial waves can be assigned the values shown in Figure B.2. Summing the partial reflected fields gives

$$\tilde{E}_{sp}^{(0)} = \left[\tilde{r}^{(0)}_{sp} + \tilde{t}^{(0)}_{sp} \tilde{r}^{(12)}_{sp} \exp(i2\tilde{\gamma}) \sum_{q=0}^{\infty} (\tilde{r}^{(10)}_{sp} \tilde{r}^{(12)}_{sp}) \exp(i2\tilde{\gamma}) \right] \tilde{E}_{sp}^{(0)}.$$  \hspace{1cm} (B.14)

Fresnel’s equations guarantee that $|\tilde{r}^{(0)}_{sp}| \leq 1$ and $|\tilde{r}^{(12)}_{sp}| \leq 1$ (for lossless and lossy media); therefore, the infinite summation in Eq. (B.14) is a convergent geometric series. Performing the summation and simplifying the result using the Stokes relations (B.10)-(B.11) gives the reflectivity as

$$\tilde{r}_{sp} = \frac{\tilde{r}^{(0)}_{sp} + \tilde{t}^{(0)}_{sp} \tilde{r}^{(12)}_{sp} \exp(i2\tilde{\gamma})}{1 + \tilde{r}^{(10)}_{sp} \tilde{r}^{(12)}_{sp} \exp(i2\tilde{\gamma})}.$$  \hspace{1cm} (B.15)

In a similar fashion, summing the partial transmitted fields gives

$$\tilde{E}_{sp}^{(0)} = \left[\tilde{t}^{(01)}_{sp} \tilde{t}^{(12)}_{sp} \exp(i\tilde{\gamma}) \sum_{q=0}^{\infty} (\tilde{r}^{(10)}_{sp} \tilde{r}^{(12)}_{sp}) \exp(i2\tilde{\gamma}) \right] \tilde{E}_{sp}^{(0)},$$  \hspace{1cm} (B.16)

which simplifies to

$$\tilde{t}_{sp} = \frac{\tilde{t}^{(01)}_{sp} \tilde{t}^{(12)}_{sp} \exp(i\tilde{\gamma})}{1 + \tilde{r}^{(10)}_{sp} \tilde{r}^{(12)}_{sp} \exp(i2\tilde{\gamma})}.$$  \hspace{1cm} (B.17)

Taking the limit as $d_i \rightarrow 0$ we obtain the following useful identities:
\[ T_{(01)}^{(02)} = \frac{T_{e_{(01)}}^{(02)} + T_{e_{(12)}}^{(02)}}{1 + T_{e_{(12)}}^{(02)} T_{e_{(01)}}^{(02)}} \]  

(B.18)

and

\[ T_{e_{(02)}}^{(02)} = \frac{T_{e_{(01)}}^{(02)} T_{e_{(12)}}^{(02)}}{1 + T_{e_{(12)}}^{(02)} T_{e_{(01)}}^{(02)}}. \]  

(B.19)

**B.4 Multilayer Films**

**B.4.1 Transfer Matrix Methods**

Although the method of summing partial waves is elegant in the case of a single layer, such calculations become unwieldy for any more layers. Owing to the linearity of the Maxwell equations (for linear media) and their boundary conditions, several transfer matrix methods have been developed to calculate the electromagnetic fields at different depths in layered media. One of the most well known formulations is the \( 2 \times 2 \) transfer matrix method for isotropic media formulated by Abelès; versions of this formulation are found in many optics textbooks and the primary literature [1-3, 5-11]. The power of this approach is exhibited by a \( 4 \times 4 \) generalization of Abelès’ method, which allows anisotropic and optically active media (e.g. chiral molecules, Faraday rotation, and cholesteric liquid crystals) to be treated [4, 12]. A lesser known \( 2 \times 2 \) transfer matrix method for isotropic media has been formulated by Hayfield and White [4, 13]. Both \( 2 \times 2 \) methods are well suited for numerical calculations of layered isotropic films. Hayfield and White’s method gives simple physical meanings to the matrix elements and analytical expressions in a particularly useful form for analysis of OI-RD data. Therefore, this formulation will be developed here.

As previously discussed, the electric field in each medium is composed of a forward propagating (+) and a back propagating (-) wave

\[ \vec{E} = \vec{E}^{(+) \parallel} + \vec{E}^{(-) \parallel} \]

\[ = \vec{E}_{i_{(+ \parallel)}}^{(+) \parallel} + \vec{E}_{i_{(- \parallel)}}^{(-) \parallel} + \vec{E}_{i_{(+) \parallel}}^{(+) \parallel} + \vec{E}_{i_{(- \parallel)}}^{(-) \parallel} \]

\[ = \vec{E}_{i_{(+) \perp}}^{(+) \perp} \hat{s} + \vec{E}_{i_{(- \perp)}}^{(-) \perp} \hat{p} + \vec{E}_{i_{(+) \perp}}^{(+) \perp} \hat{s} + \vec{E}_{i_{(- \perp)}}^{(-) \perp} \hat{p}, \]

(B.20)

where \( \hat{s} \) and \( \hat{p} \) are unit vectors perpendicular and parallel to the plane of incidence, respectively (see Appendix A for more detail). Within a layer, the \( x, y \) and \( t \) dependence is the same for both waves and polarizations, so only the \( z \) dependence will be tracked. For \( s \) and \( p \)-polarizations, \( 2 \times 1 \) column matrices are defined as

\[ \vec{E}_{i_{e_{(+) \perp}}}^{(+) \perp} (z) = \begin{bmatrix} \hat{E}_{i_{e_{(+) \perp}}}^{(+) \perp} (z) \\ \hat{E}_{e_{(+) \perp}}^{(+) \perp} (z) \end{bmatrix}. \]

(B.21)
Because of the linearity of the Maxwell equations and boundary conditions, the column matrices at any two values of \( z \) are related by a linear transformation \( \mathbf{M} \),

\[
\mathbf{M}_{sp} (z, z') \cdot \mathbf{E}_{sp} (z') = \mathbf{E}_{sp} (z).
\] (B.22)

The matrix depends upon the polarization as well as the endpoints. The linear transformation connecting the fields just above the first interface and just below the last interface is needed for computing the reflectivity and transmittivity of the film. This matrix is called the transfer matrix of the film. The \( z \) coordinate of the \((m-1)\)-th interface (see Figure B.1) is

\[
z_{o} = 0, \quad z_{n} = \sum_{q=1}^{m} d_{q}, \quad m = 1, 2, \ldots, M.
\] (B.23)

and a value immediately above the interface is denoted as \( z_{n}^- \) and immediately below the interface as \( z_{n}^+ \).

With this notation, the film transfer matrix \( \mathbf{S} \) for \( s \) or \( p \)-polarization is defined by

\[
\mathbf{S}_{sp} \cdot \mathbf{E}_{sp} (z_{n}^-) = \mathbf{E}_{sp} (z_{n}^+).
\] (B.24)

Note that the transfer matrix transforms the fields at the bottom of the film to the fields at the top of the film. Explicitly writing out the elements of the matrices as

\[
\begin{bmatrix}
\tilde{S}_{11} & \tilde{S}_{12} \\
\tilde{S}_{21} & \tilde{S}_{22}
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_{1}^0 \\
0
\end{bmatrix} =
\begin{bmatrix}
\tilde{E}_{1}^0 \\
\tilde{E}_{2}^0
\end{bmatrix}
\] (B.25)

reveals that the transmittivities and reflectivities of the film are given by

\[
\tilde{r}_{sp} = \frac{\tilde{S}_{12}}{\tilde{S}_{11}},
\] (B.26)

and

\[
\tilde{t}_{sp} = \frac{1}{\tilde{S}_{22}}.
\] (B.27)

It now remains to determine the transfer matrix for a given film. As discussed in the previous section, the phase and amplitude of the phasors change when the waves encounter an interface or propagate through a layer. Therefore, a transfer matrix \( \mathbf{I}_{sp}^{(m\rightarrow n)} \) can be associated with each interface,

\[
\mathbf{I}_{sp}^{(m\rightarrow n)} \cdot \mathbf{E}_{sp} (z_{n}^-) = \mathbf{E}_{sp} (z_{n}^+),
\] (B.28)

and a transfer matrix \( \mathbf{L}_{sp}^{(n)} \) can be associated with each layer,

\[
\mathbf{L}_{sp}^{(n)} \cdot \mathbf{E}_{sp} (z_{n}^-) = \mathbf{E}_{sp} (z_{n-1}^-),
\] (B.29)

such that the total transfer matrix can be expressed as

\[
\mathbf{S}_{sp} = \mathbf{I}_{sp}^{(01)} \cdot \mathbf{L}_{sp}^{(1)} \cdot \mathbf{I}_{sp}^{(12)} \cdot \mathbf{L}_{sp}^{(2)} \cdots \mathbf{I}_{sp}^{(M-1,M)} \cdot \mathbf{L}_{sp}^{(M)} \cdot \mathbf{I}_{sp}^{(M,M+1)}.
\] (B.30)
The interface transfer matrices can be evaluated using the superposition principle and Fresnel equations, as illustrated in Figure B.3. Comparing the waves moving away from the interface in Figure B.3, one finds that

\[
\tilde{E}_{zp}^{(i)} (z^+_n) = \tilde{P}_{zp}^{(m,n+1)} \tilde{E}_{zp}^{(i)} (z^+_n) + \tilde{P}_{zp}^{(m+1,n)} \tilde{E}_{zp}^{(i)} (z^+_n) \tag{B.31}
\]

and

\[
\tilde{E}_{zp}^{(i)} (z^-_n) = \tilde{P}_{zp}^{(m,n+1)} \tilde{E}_{zp}^{(i)} (z^-_n) + \tilde{P}_{zp}^{(m+1,n)} \tilde{E}_{zp}^{(i)} (z^-_n). \tag{B.32}
\]

Rearranging Eqs. (B.31) and (B.32) and simplifying using the Stokes relations (B.10)–(B.11) gives

\[
\tilde{I}_{zp}^{(m,n+1)} = \frac{1}{\tilde{I}_{zp}^{(m,n+1)}} \left[ \begin{array}{cc} 1 & \tilde{P}_{zp}^{(m,n+1)} \\ \tilde{P}_{zp}^{(m,n+1)} & 1 \end{array} \right]. \tag{B.33}
\]

Lastly, the layer transfer matrices can be written out using propagation factors for the plane waves, such as Eqs. (B.12) and (B.13). The result is

\[
L_{zp}^{(l)} = \begin{bmatrix} \exp(-i\gamma_n) & 0 \\ 0 & \exp(i\gamma_n) \end{bmatrix}, \tag{B.34}
\]

where

\[
\gamma_n = 2\pi n \cos \theta_n \frac{d_n}{\lambda} \tag{B.35}
\]

and \( \lambda \) is the vacuum wavelength of the plane wave.

**B.4.2 Single and Double Layer Films**

Analytical expressions for single and double layer films are easily derived using the transfer matrix method described above. For a single layer film, the transfer matrix is

\[
\tilde{S}_{zp}^{(l)} = \frac{1}{\tilde{I}_{zp}^{(l)}} \tilde{I}_{zp}^{(l)} \tilde{S}_{zp}^{(l)} \tilde{I}_{zp}^{(l)} = \frac{\exp(-i\gamma l)}{\tilde{I}_{zp}^{(l)}} \begin{bmatrix} 1 + \tilde{r}_{zp}^{(l)} \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l) & \tilde{r}_{zp}^{(l)} + \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l) \\ \tilde{r}_{zp}^{(l)} + \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l) & \tilde{r}_{zp}^{(l)} \tilde{r}_{zp}^{(2l)} + \exp(i2\gamma l) \end{bmatrix}, \tag{B.36}
\]

from which the reflectivity and transmittivity are found to be

\[
\tilde{r}_{zp} = \frac{\tilde{r}_{zp}^{(l)} + \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l)}{1 + \tilde{r}_{zp}^{(l)} \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l)} \tag{B.37}
\]

and

\[
\tilde{t}_{zp} = \frac{\tilde{t}_{zp}^{(l)} \tilde{t}_{zp}^{(2l)} \exp(i\gamma l)}{1 + \tilde{r}_{zp}^{(l)} \tilde{r}_{zp}^{(2l)} \exp(i2\gamma l)} \tag{B.38}
\]

in agreement with Eqs. (B.15) and (B.17).
For a double layer film, the transfer matrix is

\[
S_{sp} = \frac{1}{I_{sp}^{(01)} I_{sp}^{(12)} I_{sp}^{(23)}} \begin{bmatrix}
1 & \tilde{r}_{sp}^{(01)} & \tilde{r}_{sp}^{(12)} & \tilde{r}_{sp}^{(23)} \\
\tilde{r}_{sp}^{(01)} & 0 & 0 & 0 \\
\tilde{r}_{sp}^{(12)} & 1 & 0 & 0 \\
\tilde{r}_{sp}^{(23)} & 0 & -e^{i\gamma} & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & e^{-i\gamma} & 0 & 0 \\
0 & 0 & e^{-i\gamma} & 0 \\
0 & 0 & 0 & -e^{-i\gamma}
\end{bmatrix} \begin{bmatrix}
1 & \tilde{r}_{sp}^{(01)} & \tilde{r}_{sp}^{(12)} & \tilde{r}_{sp}^{(23)} \\
\tilde{r}_{sp}^{(01)} & 0 & 0 & 0 \\
\tilde{r}_{sp}^{(12)} & 1 & 0 & 0 \\
\tilde{r}_{sp}^{(23)} & 0 & -e^{i\gamma} & 1
\end{bmatrix}
\]

\[= e^{-i(\gamma + \gamma_s)} \begin{bmatrix}
1 + \tilde{r}_{sp}^{(01)} e^{i\gamma} + (\tilde{r}_{sp}^{(01)} + \tilde{r}_{sp}^{(12)} e^{i\gamma}) & \tilde{r}_{sp}^{(12)} e^{i\gamma} & \tilde{r}_{sp}^{(23)} e^{i\gamma} \\
\tilde{r}_{sp}^{(01)} + \tilde{r}_{sp}^{(12)} e^{i\gamma} & 1 + \tilde{r}_{sp}^{(12)} e^{i\gamma} & \tilde{r}_{sp}^{(23)} e^{i\gamma} \\
\tilde{r}_{sp}^{(01)} + \tilde{r}_{sp}^{(12)} e^{i\gamma} & \tilde{r}_{sp}^{(12)} e^{i\gamma} & 1 + \tilde{r}_{sp}^{(23)} e^{i\gamma}
\end{bmatrix}, \tag{B.39}
\]

from which the reflectivity and transmittivity are found to be

\[
\tilde{r}_{sp} = \left( \frac{\tilde{r}_{sp}^{(01)} + \tilde{r}_{sp}^{(12)} e^{i\gamma}}{1 + \tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}} \right) + \left( \frac{\tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}}{1 + \tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}} \right) \tilde{r}_{sp}^{(23)} e^{i\gamma}, \tag{B.40}
\]

and

\[
\tilde{t}_{sp} = \frac{\tilde{t}_{sp}^{(01)} \tilde{t}_{sp}^{(12)} e^{i(\gamma + \gamma_s)}}{1 + \tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}} + \left( \frac{\tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}}{1 + \tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}} \right) \tilde{p}_{sp}^{(23)} e^{i\gamma}. \tag{B.41}
\]

Eqs. (B.40) and (B.41) can be rearranged as

\[
\tilde{r}_{sp} = \frac{\tilde{r}_{sp}^{(01)} + \tilde{r}_{sp}^{(12)} e^{i\gamma}}{1 + \tilde{r}_{sp}^{(01)} \tilde{r}_{sp}^{(12)} e^{i\gamma}} \exp(i2\tilde{\gamma}_s), \tag{B.42}
\]

and

\[
\tilde{t}_{sp} = \frac{\tilde{t}_{sp}^{(01)} \tilde{t}_{sp}^{(12)} e^{i\gamma_s}}{1 + \tilde{t}_{sp}^{(01)} \tilde{t}_{sp}^{(12)} e^{i\gamma_s}} \exp(i2\tilde{\gamma}_s), \tag{B.43}
\]

where

\[
\tilde{r}_{sp}^{(12)} = \frac{\tilde{r}_{sp}^{(12)} + \tilde{r}_{sp}^{(23)} e^{i\gamma_s}}{1 + \tilde{r}_{sp}^{(12)} \tilde{r}_{sp}^{(23)} e^{i\gamma_s}} \exp(i2\tilde{\gamma}_s), \tag{B.44}
\]

and

\[
\tilde{t}_{sp}^{(12)} = \frac{\tilde{t}_{sp}^{(12)} \tilde{t}_{sp}^{(23)} e^{i\gamma_s}}{1 + \tilde{t}_{sp}^{(12)} \tilde{t}_{sp}^{(23)} e^{i\gamma_s}} \exp(i2\tilde{\gamma}_s). \tag{B.45}
\]
Figure B.1  Propagation Of Plane Waves In A Multilayer Film System

The arrows represent the wave vectors of the forward and back propagating planes waves.
Figure B.2  Evaluation Of The Reflectivity And Transmitivity Of A Single Layer Film Using Partial Waves

The arrows represent the wave vectors of the partial waves. The labels near the arrows indicate the values of the corresponding electric field phasors (for either s- or p-polarizations).
Appendix B

Figure B.3 Derivation Of The Interface Transfer Matrix

The arrows represent the wave vectors of the waves. The labels near the arrows indicate the values of the corresponding electric field phasors (for either either s- or p-polarizations).
References